SAE Aerospace
An SAE International Group

# AEROSPACE INFORMATION REPORT

SAE AIR1608 REV. A

Issued 1989-07
Revised 2002-04
Reaffirmed 2008-02

## Estimation of Total Error in Altimetry



### INTRODUCTION

In the field of altimetry, and more generally in the field of air data, it has been customary to specify accuracies in the form of tolerances on specific functions. For example, the specification for an altimeter or air data computer might say that for a particular test condition, the instrument would have a tolerance of ±100 ft for scale error, 50 ft for hysteresis, 50 ft for friction, 20 ft for temperature, etc. Because scale error is the most obvious, and usually the largest of the errors, mistaken assumptions have often been made that scale error alone is a sufficient measure of the accuracy of an instrument, or even of a whole system.

Some of those in the field have advocated that tolerances should be lumped; that is that test procedures should be devised so that for a particular test point, the equipment should be tested so that it would be exposed to all of the pertinent sources of error, and a single numerical limit be set on the net of all of them. This concept has been mooted for many years, but has never found full acceptance.

There remains a need for some means to know the width of the total band of error or uncertainty so that it can be stated that a particular aircraft will be within so many feet of being at its assigned altitude, and that there is good confidence (say, ±3σ or 99.7%) in the statement.

The need for this information is to be able to establish standards of vertical separation between aircraft and standards of equipment performance in order to operate safely with those vertical separation standards.



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INTRODUCTION (Continued)

In the 1950s and 60s, air carriers flying the North Atlantic were anxious for economic reasons to fly at 1000 ft, rather than 2000 ft separations. Studies were made by the Air Transport Association (ATA), and four of the resulting reports were eventually used as the bases for SAE documents (References 11, 13, 14, and 15). The International Civil Aviation Organization (ICAO) has sponsored on-going studies of the problem by its Panel on Vertical Separation of Aircraft, later renamed Review of the General Concept of Separation Panel (RGCSP). In 1982, the Federal Aviation Administration (FAA) asked the Radio Technical Commission for Aeronautics (RTCA) to form a Special Committee (SC-150) to develop a Minimum System Performance Standard (MSPS) to prepare for reduction of vertical separation to 1000 ft above Flight Level (FL) 290.

This Report suggests methods of estimating total error for four flight regimes, as originally considered in Area Navigation (RNAV) studies. More specifically, it suggests starting with the limits of probable error (i.e., tolerances) of contributing types of error and arriving at overall limits of probable error or uncertainty. It also provides for starting with known or measured contributing errors and arriving at an overall error, and for the situation where the available information consists of a mixture of tolerances and known errors.

The Traffic Alert and Collision Avoidance System (TCAS) program does not yet call for specific efforts to improve the accuracy of altimetry. "A Field Study of Mode C Altimetry Accuracy in the General Aviation Fleet" (Reference 22) shows that it is presently adequate up to 3900 ft. At high altitudes however, non-servoed pressure altimeters are less accurate than at low, and at high speeds airframe static pressure (position) errors become more significant. In the investigation of reports of apparent malfunction or false warning of TCAS equipment at high altitudes and/or speeds, where the accuracy of altimetry is suspect, the procedures of this AIR and of RTCA SC-150's "Minimum System Performance Standards for Vertical Separation above Flight Level 290" (Reference 23) should be useful.

Altitudes in this report are in terms of feet. Altitudes may some day be stated in meters or other units, but that day is further into the future than the adoption of other SI units. References 17 and 18 contain backgrounds on this question.



## 1. SCOPE:

AIR1608 ESTIMATION OF TOTAL ERROR IN ALTIMETRY proposes a method of estimating overall error of altimetry in order to provide a basis for safe vertical separation of aircraft.

## 2. REFERENCES:


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5. "Survey of Altitude Measuring Methods for the Vertical Separation of Aircraft", Gracey, Technical Note D-738, National Aeronautics and Space Administration, Langley Field, March 1961.
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8. "Survey of the Errors of Pressure Measuring Instruments in Relation to Air Traffic Separation Standards", Anderson, Technical Report 65262, Royal Aircraft Establishment, Farnborough, December 1965. (AD 478915)
9. "Specifying the Calibration of Static Pressure Systems for the Safe Use of 1000 Foot Vertical Separation Standard in North Atlantic Jet Traffic", Reich and Anderson, Technical Report 66156, Royal Aircraft Establishment, Farnborough, May 1966.
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17. "Problems in Worldwide Standardization of the Units of Height Measurement", Gilsinn, Report FAA-EM-78-2, Federal Aviation Adminstration, Washington, February 1978. (AD A051150)
18. "SI Units of Measurement in Aviation (Together with Non-Linear Units of Altitude)", Anderson, Engineering Report F-1856, United Airlines, San Francisco, September 1978. (No text; a collection of references.)

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19. "Estimation of Total Error in Altimetry", Anderson, Engineering Report F-1870, United Airlines, San Francisco, March 1979. (Draft of Society of Automotive Engineers Aerospace Information Report 1608.)

20. "Measurement of Aircraft Speed and Altitude", Gracey, Reference Publication 1046, National Aeronautics and Space Administration, Hampton, May 1980. Also published by John Wiley, New York, May 1982.

21. "Height Indication by Pressure Altimeters", Anderson, Engineering Report F-1588, United Airlines, San Francisco, July 1980.

22. "A Field Study of Mode C Altimetry Accuracy in the General Aviation Fleet", Cohen, Report MTR-86W231, Mitre Corporation, McLean, March 1987.

23. "Minimum System Performance Standards for Vertical Separation Above Flight Level 290", Special Committee 150, Radio Technical Commission for Aeronautics, Washington, Fifth Draft, January 1988. [More drafts expected before publication as an RTCA DO- ____document.]

## 3. GROUPING OF ERRORS:

Cursory discussions of accuracy of altimetry are usually concerned with the errors peculiar to one aircraft and its equipment, or in the case of collision probability studies, two aircraft. While this is not invalid, neither is it complete. There are additional sources of error that are common to all aircraft in a locality, others that are common to a type of aircraft, and still others related to the manner of operation of the aircraft.

### 3.1 Errors Common to all Aircraft in a Given Locality:

Aircraft below 18,000 ft and receiving their QNH altimeter setting number from a single source are all equally dependent on the accuracy of that source. If there are errors in the QNH transmitted, all aircraft using it will be flying too low or too high by the same amount. This may increase the risk of long or short landings or of hitting obstructions on the ground, but should not increase the risk of collision between two aircraft.

### 3.2 Errors Pertaining to Aircraft of a Type:

One of the steps on certification of an aircraft type is flight calibration of the static system of one or more of the first few aircraft. It has been recommended that at least three aircraft be calibrated (Reference 12). It is usual that the flight test results are a series of data points with some scatter, and that a fair curve, or family of fair curves is drawn through the scatter field. The fair curve then is published in the government-approved Flight Manual as applicable to all aircraft of that type. Subsequent aircraft of the type will thus be subject to whatever errors or uncertainties there were in the original calibration.

3.2 (Continued):

Where the airframe builder has not been able to find a static source of negligible error, he should provide the aircraft with an automatic correction system, such as might be included in a central air data computer. There are no rules defining how large an error is negligible. One airframe builder might consider up to 75 ft of altitude error negligible, while another might choose to neglect 200 ft.

Until automatic altitude reporting for traffic control became a requirement, it was feasible to post near the altimeter a correction card. Altitude reporting regulations, however, require that the altitude reported digitally via the Air Traffic Controller Transponder agree (on a 1013.2 mb/29.92 in Hg basis) with the altitude shown by the altimeter within 125 ft. In effect, this precludes the use of correction cards and requires either that there be an automatic correction system or that the error be asserted to be negligible.

3.3 Errors Pertaining to an Individual Airframe:

In an aircraft with small flush static pressure port fittings set in the skin, there may be minor variations of skin contour near the ports as compared with the aircraft used in the original flight calibration and type certification. These may be due to manufacturing variations and/or to subsequent damage and could affect calibration.

Where Pitot-static probes or hard flush static port plates (Reference 12) are used instead of small flush ports in the skin, surface condition is more readily controllable, although probes are exposed to surface deterioration, damage, and misalignment which could affect calibration.

3.4 Errors Pertaining to an Individual Aircraft Set of Equipment:

Any set of equipment, whether a pressure-operated altimeter or an air data computer driving an electric altitude indicator, will be subject to scale, temperature, friction and other errors. In computers which apply a static pressure correction, errors of Mach will affect accuracy of altitude output, particularly at high speed. Where the atmospheric pressure is measured in a computer and the altitude is transmitted electrically to an indicator, there may be error in the servo loop; in the absence of gross malfunction, this should not exceed 5 ft and is disregarded in this report.

Aircraft equipment will also be subject to the errors of the shop equipment used to calibrate it. In this report only the accuracy of the barometer or other absolute pressure standard is considered.

3.5 Flight Technical Error:

Errors of pilot interpretation of vertical guidance instrumentation, pilot operation of aircraft vertical controls and deviations caused by aircraft response characteristics are often lumped together as flight technical error. For the purposes of this report, reading error and altitude hold error are considered separately.

3.5.1 The least increment of most altimeter readouts is 20 ft. In level flight, reading errors should not exceed this. (Some altimeter displays are said to be susceptible to misreading by 1000 or 10,000 ft, but that problem is considered to be outside the scope of this report.) In ascent and descent the errors are likely to be larger than 20 ft, and lagging. Thus, reading errors would be treated as rectangularly distributed in level flight, but as plus offset in descent.

The plus offset could be aggravated in the situation where approach and landing are done on a QNH basis. Here the pilot must repeatedly read the altitude above sealevel and mentally subtract from it the field elevation in order to get the height of the airplane above the field. The errors and lags of the subtraction process can be eliminated by operating the altimeter on a QFE basis. Definitions and derivations of QNH and QFE are given in Reference 1. When QFE in mb or in Hg is set on the baro scale of the altimeter, the instrument will display height above the field rather than altitude above sealevel. At touchdown it will read zero. Reference 21 gives a method whereby the QNH reading from the airport can be converted to QFE by the pilot.

3.5.2 Most discussions of altitude hold performance, whether of the pilot or autopilot, deal with the calm air conditions. In turbulence, altitude hold errors could be up to, say, four times larger than in calm air.

## 4. FLIGHT REGIMES:

Aircraft are capable of operation in a large variety of combinations of speeds and altitude, constant or changing. For the purposes of this report, four typical situations are considered.

### 4.1 Descent and Landing:

Descent, from say 8000 ft, and landing is a critical situation for altimetry because there may be ground obstructions to be cleared. Where no glideslope, radar altimeter, or other radio-type landing aid is available, altimetry may determine when the pilot begins flareout.

### 4.2 Descent and Holding:

Descent from cruise altitude is often interrupted by a period of holding at medium altitude while awaiting clearance to approach and land. Numbers of aircraft are often stacked with nominal 1000 ft vertical separations.

### 4.3 Cruise or Holding:

The range of altitudes from say, 19,000 to 30,000 ft are typical cruise altitudes for pressurized propeller aircraft and occasionally jet aircraft. Rarely, holding patterns extend up into this range.

### 4.4 Cruise:

Altitudes 31,000 and above are typical for long-range cruise of jet aircraft.

4.5 Climbout:

The climbout after takeout is not considered in this report because clearance from the ground and its obstructions is increasing, and aircraft are diverging horizontally.

## 5. TYPICAL ERRORS:

Table 1 lists errors which might be typical for five groups of errors and in four flight regimes. They are assumptions, and the basis for most of the assumptions are stated.

CAUTION: The numerical values used only illustrate a proposed method of estimating total error of altimetry. It must not be inferred that they apply to any specific aircraft or equipment.



## 6. ESTIMATION OF TOTAL ERROR:

6.1 Estimate from Tolerances:

6.1.1 Table 1 lists magnitudes and signs of errors assumed to be typical for the purposes of this report. It also gives worst-case totals. Fortunately, in actual practice, total errors are extremely unlikely to reach worst-case magnitudes. This comes about because component errors are seldom all at their maxima and because some cancel others.

6.1.2 It is generally accepted that the total error can be ascertained to a given confidence level by the root-sum-square method. The standard deviation, σ, is given by the square root of the sum of the square of the individual errors. Three times this on either side of the mean gives the total of the distributed errors to a 99.7% confidence level. A detailed treatment of this procedure can be found in Reference 2.

Individual errors are treated differently according to their distribution, thus:

$$\sigma = \sqrt{\left(\frac{E_{n_1}}{3}\right)^2 + \left(\frac{E_{n_2}}{3}\right)^2 + \ldots\left(\frac{E_{r_1}}{\sqrt{3}}\right)^2 \ldots} \qquad \text{(Eq. 1)}$$

where:

$E_n$ = error with normal distribution
$E_r$ = error with rectangular distribution

TABLE 1 - Typical Errors

| Error Group | Error | Flight Regime: SL - 8000, Descent & Landing | 9000 - 18,000, Descent & Holding | 19,000 - 30,000, Cruise or Holding | 30,000 - 40,000, Cruise |
|---|---|---|---|---|---|
| Common to Locality | QNH | ±25 | ±25 | 0 (a) | 0 (a) |
| Common to Aircraft Type | Static Calibration (b) | ±45 (b) | ±50 | ±50 | ±50 |
| Individual Airframe | Deviation from Static Calibration | 0 (c) | ±20 | ±30 | ±50 |
| Individual Set of Aircraft Equipment | Barometry (d) | ±5 | +10 | ±20 | ±30 |
| | Scale (e) | ±42 | ±70 | ±120 | ±150 |
| | Temperature (f) | ±8 | ±14 | ±24 | ±30 |
| | Static Correction | 0 (h) | ±30 (g,i) | ±30 (g,i) | ±30 (g,i) |
| | Dynamic Friction (j) | +25 | +25 | ±25 | ±50 |
| | Static Friction (j) | +25 | +25 | ±36 | ±50 |
| | Drift | 0 (h) | 0 (h) | +40 | +40 |
| | Hysteresis or After Effect | +30 | +48 | 0 (h) | 0 (h) |
| | Baro | ±30 (k) | +25 (l) | 0 (m) | 0 (m) |
| | Balance | ±20 (k) | ±27 (l) | 0 (m) | 0 (m) |
| Flight Technical | Altitude Hold | 0 (n) | 0 (n) | ±50 (o) | ±75 (o) |
| | Reading (p) | +40 (p) | +40 (p) | ±20 (q) | ±20 (q) |
| | Totals (Worst Case) | +295<br>-175 | +409<br>-271 | +445<br>-405 | +575<br>-535 |

TABLE 1 (Continued)

NOTE:

The assumptions and some comments upon them follow. Some values are interpolated.

a. Not applicable. At and above 18,000 ft altimeters are set to 1013.2 mb (29.92 in Hg).

b. Residual error of type calibration. For the approach and landing condition, limited by Federal Aviation Regulations to 30 ft/100 kt.

c. Deviation of static port surface conditions from standard has less influence at low speed.

d. Errors of absolute pressure standard used to calibrate aircraft instrument.

e. Greatest error in altitude range.

f. Instrument 20 °C above calibrating temperature.

g. If the airframe builder and/or the operator consider a known error to be too small to require automatic correction, it should nevertheless be entered and treated as an offset error.

h. Negligible in this range.

i. Error of subsystem applying correction for 200 ft static pressure error; 15% of 200.

j. Non-servoed type instrument with other than counter-pointer or counter-drum-pointer display. Dynamic and static friction cannot be at maximum at the same time; only the greater of the two is included.

k. Baro mechanism rotated to 846.2 mb (24.99 in Hg) for QFE landing at 4900 ft.

l. Baro correction mechanism between 954.9 and 1049.1 mb (28.20 and 30.98 in Hg).

m. At and above 18,000 ft the baro mechanism is at 1013.2 mb (29.92 in Hg), the primary calibrating position.

n. Not applicable.

o. Autopilot altitude-hold engaged; calm air. Manual control or turbulent air would increase error.

p. Reading error in descent is likely to be of plus sign and of greater magnitude than in level flight.

q. Least increment of altitude dial.

6.1.2 (Continued):

In the present examples most of the errors are considered to have normal (Gaussian) distribution and are divided by three. The exceptions are:

The distribution of scale errors of unservoed pressure altimeters should be taken as rectangular, rather than normal.

Reading error in level flight is taken as having rectangular distribution, and the magnitude of 20 ft is assumed because that is the least increment of most altimeter readouts. Rectangularly distributed errors are divided by $\sqrt{3}$.

In cruise drift is only in the plus direction. In the descent situation, friction and hysteresis errors are only in the plus direction, and reading errors are likely to be on the high side. Then friction, hysteresis and reading errors are considered as offset errors. The sum of the means of these errors (or half of one error if only one is applicable) is added to the 3 $\sigma$ value of the distributed errors.

Where the estimate is based mostly or entirely on tolerances rather than known errors, it would be better to refer to the result as an uncertainty rather than an error.

6.2 Known Errors:

6.2.1 Where the magnitude and sign of each contributing error is known, the total error is simply the net or algebraic sum of the contributing errors.

6.3 Worksheets:

6.3.1 Figure 1 is a blank worksheet for estimating total error. One sheet is needed for each flight regime. It will be noted that there is space for some of the same component errors in more than one location. For example, a friction error not known but estimated from tolerances would be shown among the normally distributed errors for the level flight condition, or as a plus offset error for the descent condition. If the actual value were known, it would instead be shown among the known errors. In any case, and one type of error is shown in only one place.

Residual errors of static pressure correction estimated from tolerances would be shown as normally distributed, plus and minus. Known static pressure errors left uncorrected would be of one sign only.

6.3.2 Figure 2 is a worksheet filled in with values from the second column of Table 1. The resulting error/uncertainty is an estimate from tolerances.

6.4 If the calculation is carried out for each of the four flight regimes, a table of error limits can be prepared which is not of worst-case totals, but of limits of probable error to a 3 $\sigma$ confidence level. Table 2 shows the results for the four flight regimes using the assumptions of Figure 1.

Note that these numbers are illustrative only, and do not apply to any specific aircraft or equipment.

TABLE 2 - Limits of Probable Error Estimated from Tolerances

| | Flight Regime | | | |
|---|---|---|---|---|
| | SL - 8000 | 9000 - 18,000 | 19,000 - 30,000 | 31,000 - 40,000 |
| | Descent & Landing | Descent & Holding | Cruise or Holding | Cruise |
| Limits of Probable Error | +122<br>-28 | +161<br>-49 | +177<br>-137 | +218<br>-178 |
| Width of Probable Error Band | 150 | 210 | 314 | 396 |



7. INCIDENT INVESTIGATION:

In the course of investigating a near-miss of collision between aircraft, or an apparently erroneous fly up or fly down signal from a TCAS, it may become important to know the actual altitude as well as the indicated altitude and that being transponded by the Mode C reporting system. Where the altimeters and/or air data computers can be removed from the aircraft in operable condition for calibration in the shop, actual readings can be obtained. Likewise actual errors of shop standards, etc. can be obtained.

Such an investigation would consist largely of adding and subtracting measured errors. Numerical values for some components of overall altimetry error, such as deviation from static calibration (airframe static pressure correction) and flight technical error are not likely to be readily available. In this case the probable error limits of unmeasured error components, as estimated in Section 6 of this report, could be used in conjuction with the net measured error to arrive at a probable altitude.

Figure 3 is an example of a worksheet for an incident in which the airplane equipment scale, temperature, friction and drift errors were measured, calibrating barometer error was found by comparison with a primary standard and airplane static calibration error was known from the Flight Manual but not corrected by the equipment. The other errors were estimated from tolerances.

For investigation of incidents at high speeds and altitudes, the somewhat more comprehensive procedures of "Estimation of Altimetry System Error" should be referred to. It is an Appendix of "Minimum System Performance Standards for Vertical Separation Above Flight Level 290" (Reference 23)

Aircraft ____________

Flight Regime ____________

Air Data Comp ____________

Altimeter ____________

Estimator ____________

Date ____________

| NORM DIST ERROR TOL | Feet | ÷ 3 | Square | |
|---|---|---|---|---|
| QNH | | | | |
| Static Calib | | | | |
| Dev Static Calib | | | | |
| Barometry | | | | |
| Scale* | | | | |
| Temperature | | | | |
| Static Corr | | | | |
| Dyn Friction | | | | |
| Stat Friction | | | | |
| Baro Coord | | | | |
| Balance | | | | |
| Altitude Hold | | | | |
| | | | | |
| Subtotal, Norm Dist | | | | ± |

| RECT DIST ERROR TOL | Feet | ÷ √3 | Square | |
|---|---|---|---|---|
| Reading | | | | |
| Scale ** | | | | |
| Subtotal Rect | | | | ± |
| Subtotal Norm & Rect | | | | ± |
| $\sqrt{\ \ }$ = $\sigma$ | | | | ± |
| 3$\sigma$, Norm & Rect | | | | ± |

| OFFSET ERROR TOL | Feet | Mean | | |
|---|---|---|---|---|
| Dyn Friction | | | | |
| Stat Friction | | | | |
| Drift | | | | |
| Hyst/Aft Eff | | | | |
| Reading | | | | |
| | | | | |
| Subtotal, Offset | | | + | - |
| Subtotal, Est from Tol | | | + | - |

| KNOWN ERROR | Feet | | |
|---|---|---|---|
| QNH | | | |
| Static Calib | | | |
| Dev Static Calib | | | |
| Barometry | | | |
| Scale | | | |
| Temperature | | | |
| Static Corr | | | |
| Dyn Friction | | | |
| Stat Friction | | | |
| Baro Coord | | | |
| Balance | | | |
| Altitude Hold | | | |
| Drift | | | |
| Hyst/Aft Eff | | | |
| Reading | | | |
| | | | |
| Subtotal, Known | | + | - |
| TOTAL, LIMITS | | + | - |
| TOTAL, BAND | | | |

*ADC or servoed altimeter.
**Unservoed altimeter.

FIGURE 1 - Blank Worksheet